Kinematic Planning for Mobile Manipulators Using Optimal Control

Mayank Mittal

* Giftthaler, Markus, et al. "Efficient kinematic planning for mobile manipulators with non-holonomic constraints using optimal control." *2017 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2017.

Motivation

- Efficiently design controllers that satisfies non-holonomic constraints

$$g(\mathbf{q}, \dot{\mathbf{q}}, t) = 0$$

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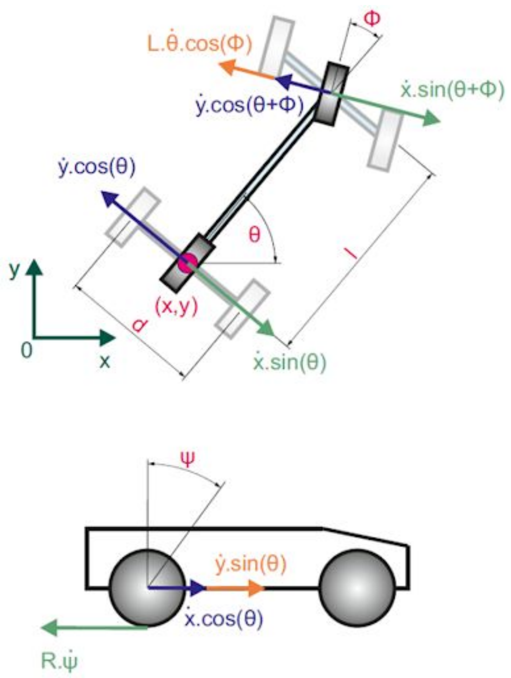


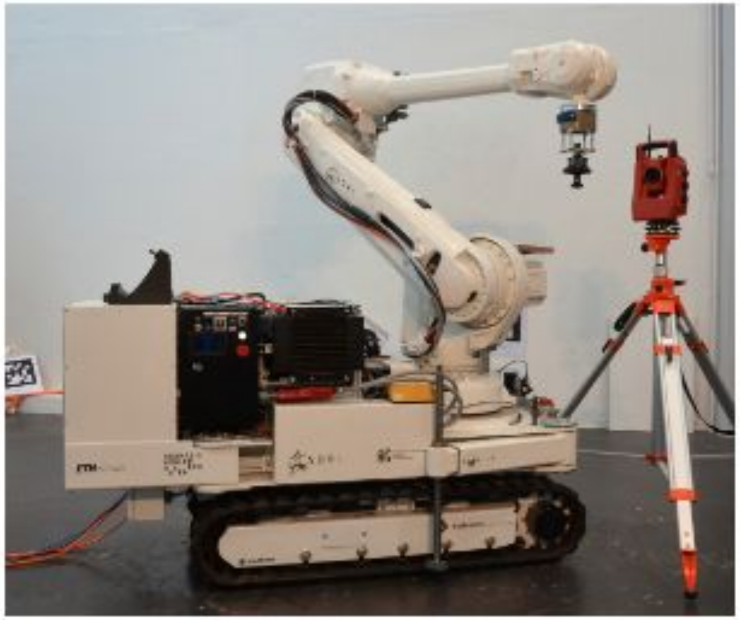
Example: Car-like robot

Assuming pure rolling contact:

$$-\dot{x}\sin\theta + \dot{y}\cos\theta = 0$$

Motivation

- Operational space tracking for mobile manipulation



In-Situ Fabrication 1

In-Situ Fabrication 2

Motivation

- Most commercially available robots are position/velocity controlled

Kinematic planning

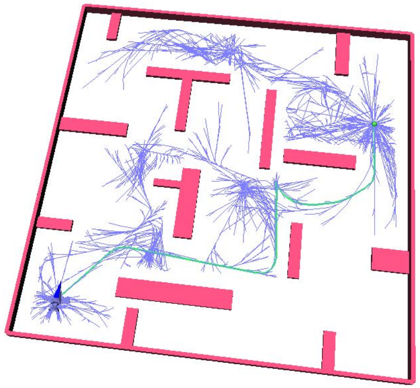
(i.e. dynamics of system is neglected)

Motivation

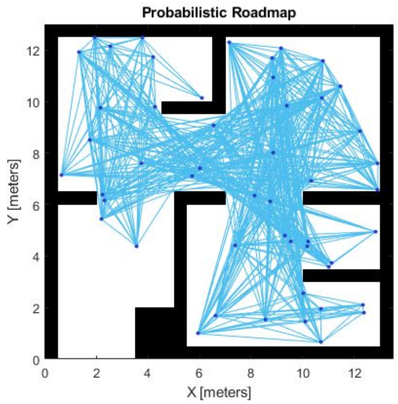
- Most commercially available robots are position/velocity controlled

Kinematic planning

Prior works produce kinematically feasible solutions but without optimality guarantees



RRT



PRM

Contributions

- Introduces ‘constrained SLQ’ to kinematic planning
  - Kinematic feedback laws are compliant with constraints
  - Does not suffer from issues in other algorithms that employ inverse kinematics (such as when approaching singularities)

^ Constrained Sequential Linear Quadratic Optimal Control

Outline

- Primers:
  - Sequential Quadratic Programming
  - Constrained SLQ
- Algorithm Overview
  - Formulation of planning and control problem
  - Receding Horizon Control Setup
- Experiments & Results

Sequential Quadratic Programming

- Used commonly for constrained non-linear optimization problems (NLPs)

$$\begin{aligned} &\text{minimize} \quad f(x) \\ &\text{over} \quad x \in \mathbb{R}^n \\ &\text{subject to} \quad h(x) = 0 \\ &\qquad\qquad\quad\;\; g(x) \leq 0 \end{aligned}$$

Linear / quadratic function

Affine functions

- Iterative procedure which models NLP for given iterate as a Quadratic Programming (QP) subproblem

Sequential Quadratic Programming

- Formulation of QP subproblem for current x^k iterate
  - Local quadratic approximation of objective f :

$$f(x) \approx f(x^k) + \nabla f(x^k)(x - x^k) + \frac{1}{2}(x - x^k)^T H f(x^k)(x - x^k)$$

  - Local affine approximations of const

$$g(x) \approx g(x^k) + \nabla g(x^k)(x - x^k)$$

$$h(x) \approx h(x^k) + \nabla h(x^k)(x - x^k)$$

Sequential Quadratic Programming

- Formulation of QP subproblem for current x^k iterate
  - Set:

$$d(x) := x - x^k \quad , \quad B_k := Hf(x^k)$$

  - QP subproblem:

$$\begin{aligned}
&\text{minimize} \quad \nabla f(x^k)^T d(x) + \frac{1}{2} d(x)^T B_k d(x) \\
&\text{over} \quad d(x) \in \mathbb{R}^n \\
&\text{subject to} \quad h(x^k) + \nabla h(x^k)^T d(x) = 0 \\
&\qquad\qquad\quad\; g(x^k) + \nabla g(x^k)^T d(x) \leq 0 \, ,
\end{aligned}$$

Constrained SLQ

- Class of dynamic programming algorithm
  - Scales linearly with optimization horizon problem
  - Designs both feedforward plans and feedback controller

- Solves n ntrol problem

$$\min_{\mathbf{u}(\cdot)} \left\{ \Phi(\mathbf{x}(t_f)) + \int_{t_0}^{t_f} L(\mathbf{x}, \mathbf{u}, t)\,dt \right\}$$

subject to

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \qquad \mathbf{x}(t_0) = \mathbf{x}_0$$

$$\mathbf{g}_1(\mathbf{x}, \mathbf{u}, t) = 0 \qquad \mathbf{g}_2(\mathbf{x}, t) = 0$$

Constrained SLQ

$$\min_{\mathbf{u}(\cdot)} \left\{ \Phi(\mathbf{x}(t_f)) + \int_{t_0}^{t_f} L(\mathbf{x}, \mathbf{u}, t) dt \right\}$$

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Time-varying linear feedback control law

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_{ff}(t) + \mathbf{K}(t)\mathbf{x}(t)$$

In each iteration:

1. Approximate above with local QP subproblem
2. Solve using Riccati-based approach

Constrained SLQ

Algorithm 1 SLQ Algorithm

Given

- Mode switch sequence and switching times
- Cost function (4) and system dynamics (5)

Initialization

- Initialize the controller with a stable control law, $\mathbf{u}_0(\cdot)$

repeat

- Forward integrate system dynamics: $\{\bar{\mathbf{x}}(z), \bar{\mathbf{u}}(z)\}_{z=0}^{I}$,
- Compute the LQ approximation of the problem along the nominal trajectory, (7) and (8).
- Solve the final value differential equations, (9-11).
- Line search for the optimal α with policy (17)
- Update control law: $\mathbf{u}^*(z, \mathbf{x}) = \bar{\mathbf{u}}(z) + \alpha^* \mathbf{l}(z) + \mathbf{L}(z)\delta\mathbf{x}$

until $\|\mathbf{l}(\cdot)\|_2 < l_{min}$ or maximum number of iterations

Farshidian, Farbod, et al. "Sequential linear quadratic optimal control for nonlinear switched systems." *IFAC-PapersOnLine* 50.1 (2017): 1463-1469.

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Kinematic Planning and Control Problem

State $\mathbf{x} = \mathbf{q}$

Input $\mathbf{u} = \dot{\mathbf{q}}$

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Kinematic Planning and Control Problem

State $\mathbf{x} = \mathbf{q}$

Input $\mathbf{u} = \dot{\mathbf{q}}$

$$\int_{t_0}^{t_f} \mathbf{u}(t)^\top \mathbf{R} \mathbf{u}(t) dt + (\mathbf{x}(t_f) - \mathbf{x}_r)^\top \mathbf{Q}_f (\mathbf{x}(t_f) - \mathbf{x}_r)$$

Desired terminal state

$$\min_{\mathbf{u}(\cdot)} \left\{ \Phi(\mathbf{x}(t_f)) + \int_{t_0}^{t_f} L(\mathbf{x}, \mathbf{u}, t) dt \right\}$$

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Covers all non-holonomic constrai

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Covers holonomic operational-spac e tracking constraints

E.x. $\mathbf{g}_2(\mathbf{q}, t) : \quad \mathbf{r}_{ee}^{frw} = \mathbf{r}_{ee,\mathbf{ref}}^{frw}(t, \mathbf{q})$

Covers all non-holonomic constrai

$$\mathbf{g}_1(\mathbf{q}, \dot{\mathbf{q}}, t) = 0$$

Receding Horizon Optimal Control Setup

Optimal Control Solver

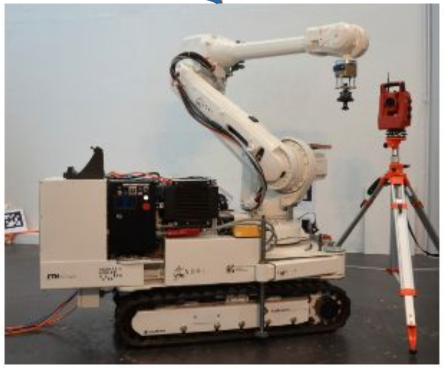
Slow (~20Hz)

$\mathbf{K}^*(t), \mathbf{u}^*_{ff}(t)$

Closed-loop Controller

Fast (~250 Hz)

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}^*_{ff}(t) + \mathbf{K}^*(t)\mathbf{x}(t)$$



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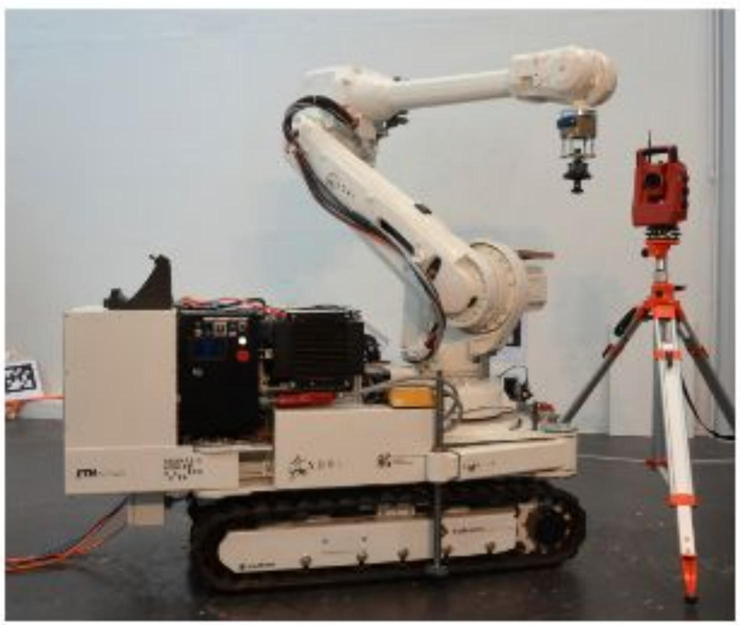
Experiments

- IF1: Tracked Mobile Manipulator
- IF2: Legged/wheeled mobile robot with DoF 26



In-Situ Fabrication 1

In-Situ Fabrication 2

Experiments: IF2

Wheel Constraints:

- Rolling condition
- Wheel should not lift off of the ground

$$\mathbf{g}_1(\mathbf{q}, \dot{\mathbf{q}}, t) \;:=\; \mathbf{v}_P^{frW} \;=\; 0$$

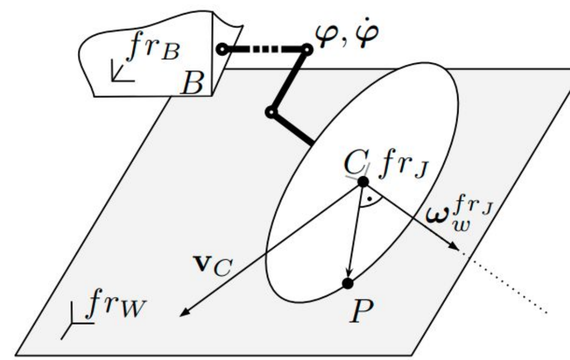
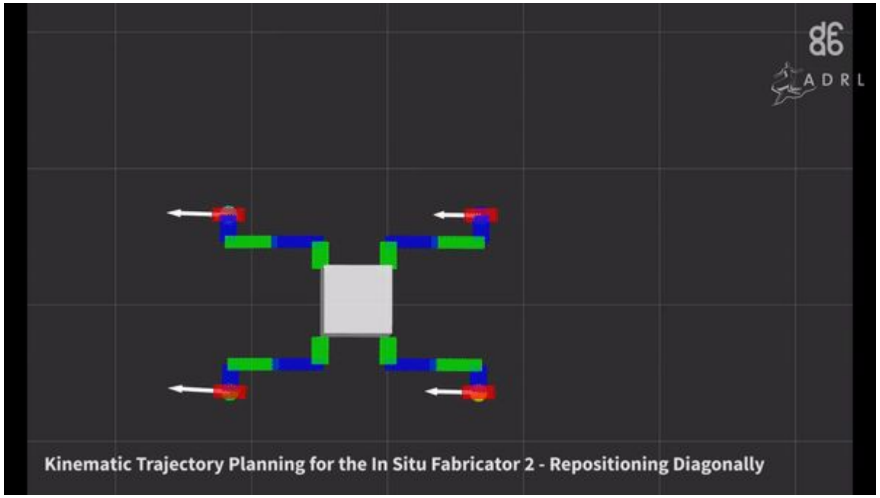


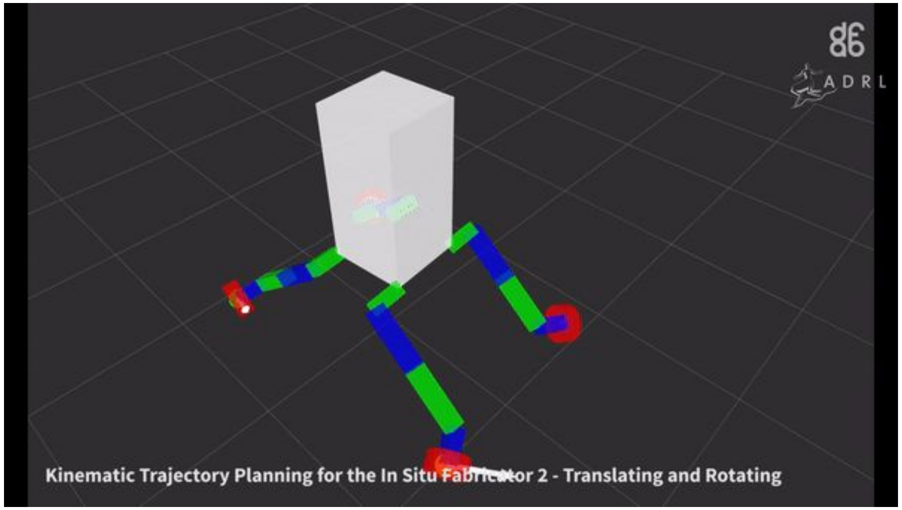
Fig. 2. Sketch of an ideal wheel attached to the robot's trunk through an arbitrary serial chain of joints and links.

$$\mathbf{v}_P^{frW} = \mathbf{R}_{frB}^{frW}(\boldsymbol{\theta}) \cdot \left(\mathbf{v}_B^{frB} + \mathbf{v}_P^{frB} + \boldsymbol{\omega}_B^{frB} \times \mathbf{r}_{BP}^{frB}\right)$$

$$\begin{bmatrix}\boldsymbol{\omega}_C^{frB\top} & \mathbf{v}_C^{frB\top}\end{bmatrix}^\top = \mathbf{J}_C^{frB}(\varphi) \cdot \dot{\varphi}$$

Experiments: IF2

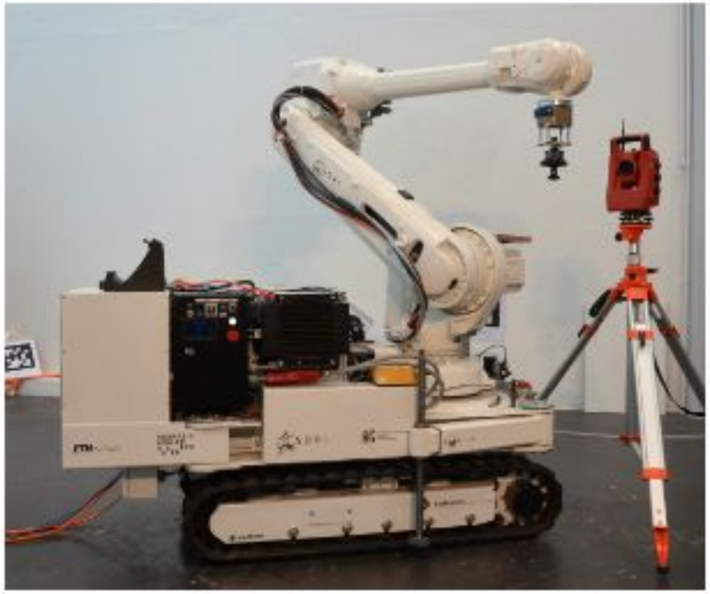




Robot needs to move to location (1, 1)

Robot has to translate 1m away and rotate trunk by 180 degrees

Experiments: IF1



Tracks simplified to two -wheel model

$$\dot{y}_c \cos\theta - \dot{x}_c \sin\theta - \dot{\theta} d = 0$$

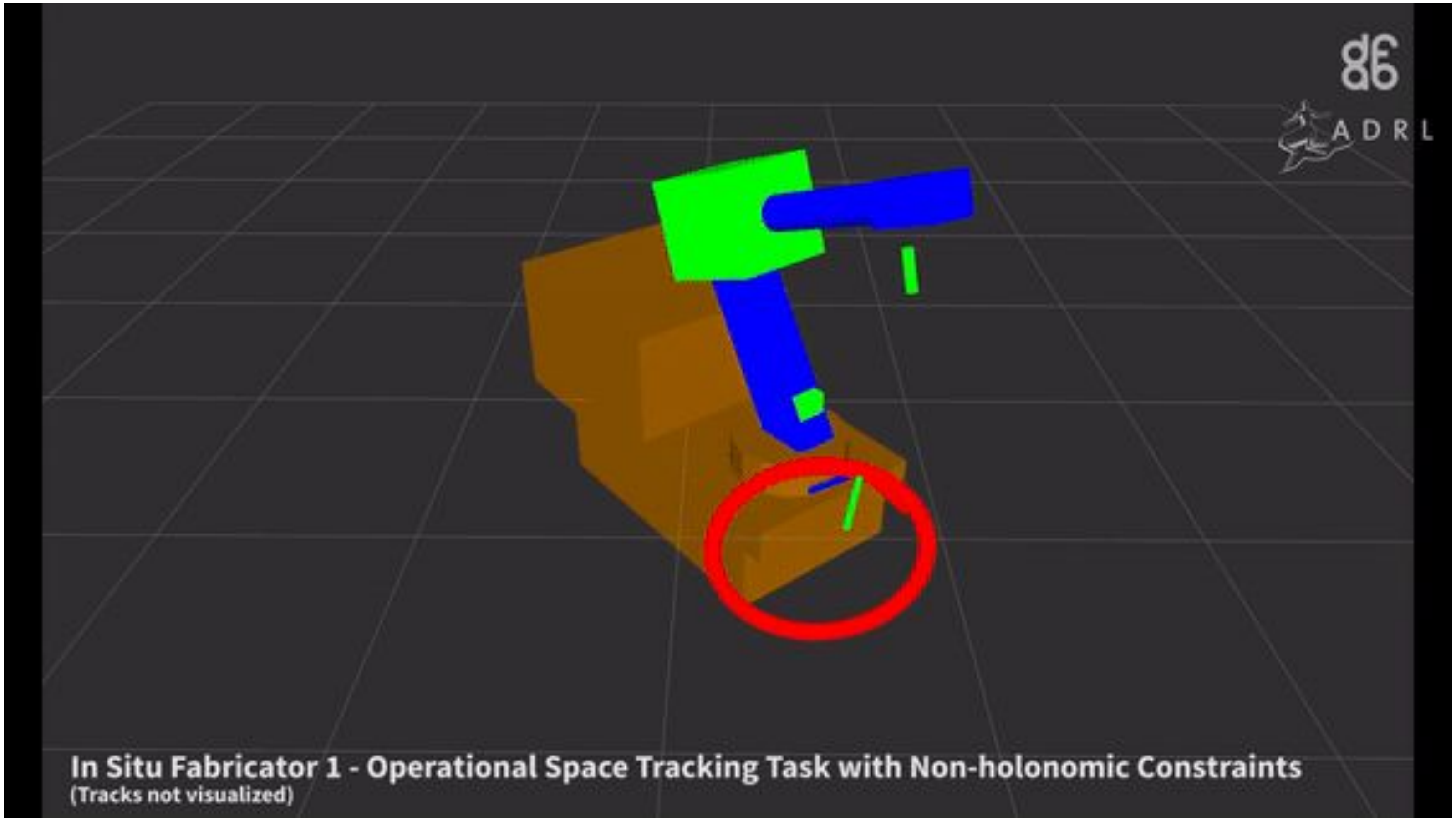
Operational-space tracking constraint

$$\mathbf{g}_1(\mathbf{q}, \dot{\mathbf{q}}, t): \quad \mathbf{v}_{ee}^{frw} = \mathbf{v}_{ee,\mathbf{ref}}^{frw}(t, \mathbf{q}, \dot{\mathbf{q}}) \quad \text{or}$$

$$\mathbf{g}_2(\mathbf{q}, t): \quad \mathbf{r}_{ee}^{frw} = \mathbf{r}_{ee,\mathbf{ref}}^{frw}(t, \mathbf{q}) \; .$$

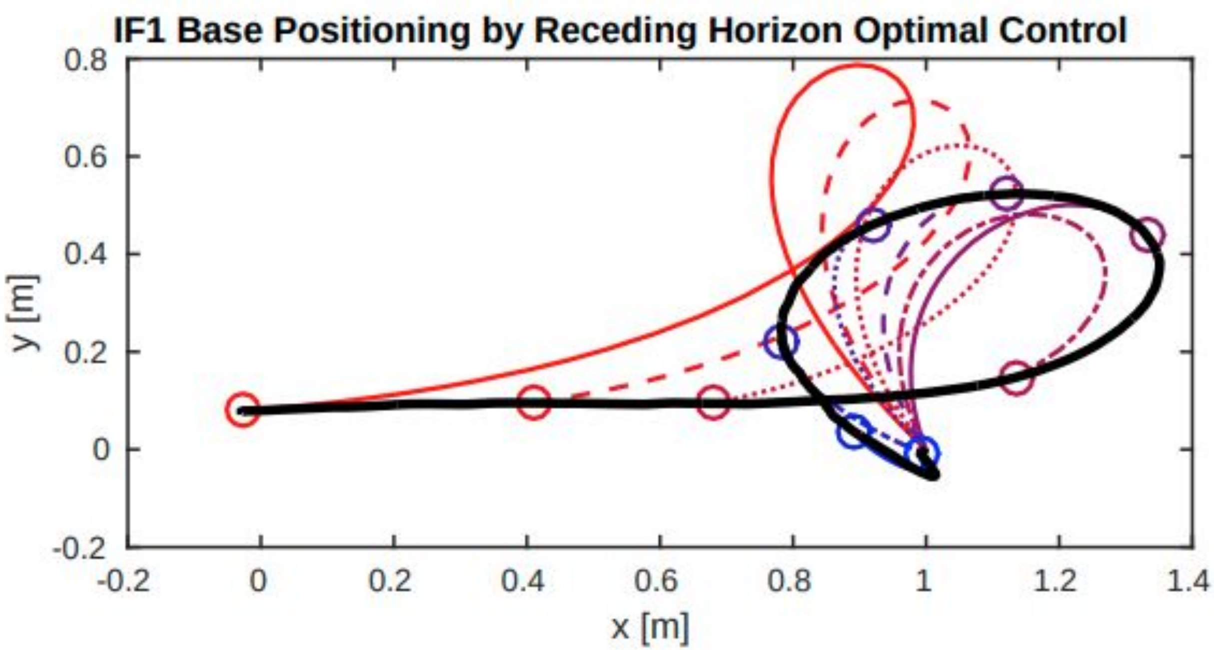
Note: Reference positions and velocities trajectories need to be continuous and differentiable.

Experiments: IF1



End-effector needs to follow given task-space trajectory

Experiments: IF1



Computed paths for base repositioning/ reorientation task using RHOC

Remarks

- Proposed method has linear time complexity in local regime w.r.t. time horizon
  - Note: not intended to compete with planners in cluttered environments
- Uses adaptive step-size ODE solver
- Achieves 50-100 Hz replanning rates (for IF2) on single core CPU

Remarks

Ongoing usages of constrained SLQ

Grandia, Ruben, et al. "Feedback MPC for Torque-Controlled Legged Robots." *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2019)*. 2019.

Minniti, Maria Vittoria, et al. "Whole-body mpc for a dynamically stable mobile manipulator." *IEEE Robotics and Automation Letters* 4.4 (2019): 3687-3694.

Thank you for your attention!